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Twist symmetry and classical solutions in open string field theory

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Abstract: We construct classical solutions of open string field theory which are not invariant under ordinary twist operation. From detailed analysis of the moduli space of the solutions, it turns out that our solutions become nontrivial at boundaries of the moduli space. The cohomology of the modified BRST operator and the CSFT potential evaluated by the level truncation method strongly support the fact that our nontrivial solutions correspond to the closed string vacuum. We show that the nontrivial solutions are equivalent to the twist even solution which was found by Takahashi and Tanimoto, and twist invariance of open string field theory remains after the shift of the classical backgrounds.

Keywords: String Field Theory, D-branes, Tachyon Condensation.

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Contents

1. Introduction

Nowadays, string field theory (SFT) turns out to be one of promising candidates for the nonperturbative formulation of string theory. Attractive features of SFT — such as background independence and local gauge symmetry — enable us to explore the moduli space and classical (or quantum) dynamics of string theory. In particular, the dynamics of unstable D-brane systems have been extensively investigated in terms of open SFT [1, 2] and all results obtained up to now support Sen's conjecture [3]. Furthermore, analysis of several systems involving closed string tachyons also have been developing [4].

In recent few years, a class of analytic classical solutions of cubic SFT (CSFT) [5] have been extensively investigated [6]–[13]. The solutions — *universal solutions* [6] — are Lorentz invariant, universal[1] and conjectured to be candidates for the closed string vacuum where there are no open strings. Each solution is labeled by a function $g(w)$ which obeys (i) BPZ invariance, (ii) midpoint constraint and (iii) hermiticity. Therefore, classification of the solutions reduce to a problem of classification of the functions satisfying above three conditions.

[1]This means that solutions are independent of details of a boundary CFT used there [14].

A key feature of the moduli space of the solutions is distributions of zeros of $g(w)$ on the unit disk. In general, it is expected that when zeros are on the unit circle, a solution becomes nontrivial, while when all of them are inside unit disk, a solution is trivial and pure gauge. Such feature of zeros was confirmed for particular cases [6, 7, 11] and proved for the general case of even polynomial functions in a systematic way [10].

After removing trivial solutions from the space of $g(w)$, there remain a lot of nontrivial solutions. If all of them correspond to unique closed string vacuum, they must be equivalent each other through gauge transformations. In fact, there are strong evidences [12] that nontrivial solutions considered in [7] are equivalent through the 'global' part of the local gauge symmetry generated by $K_{2m} = L_{2m} - L_{-2m}$. Such result allows us to interpret tachyon condensation as a dynamical symmetry breaking concerned with the CSFT potential.

From these results, it is natural to ask whether classification of the whole moduli space of universal solutions is possible. As mentioned before, the subspace of solutions labeled by even polynomial functions can be analyzed systematically [10]. On the other hand, a case involving an odd part of $g(w)$ is more complicated than the even case owing to the fact that the midpoint condition $g(\pm i) = 1$ becomes nontrivial in such case. For such reason, the odd part have never been considered earlier.

Besides the problem of classification of the solution mentioned above, introducing an odd part of $g(w)$ poses some questions to us. A decomposition of $g(w)$ according to worldsheet parity is given by

$$g(w) = g_+(w) + g_-(w)\,, \tag{1.1}$$

where $g_\pm(w)$ are even and odd parts of $g(w)$ which satisfy $g_\pm(-w) = \pm g_\pm(w)$ respectively. An important fact is that (1.1) yields a decomposition of the corresponding classical solution

$$\Psi = \Psi_+ + \Psi_-\,, \tag{1.2}$$

where Ψ_+ and Ψ_- are *twist*[2] even and odd parts of the universal solution associated with $g(w)$, respectively. Thus, inclusion of an odd part of $g(w)$ yields an odd component of a classical solution. Such case have never been considered in any attempts to find the closed string vacuum — level truncation in Siegel gauge [15−17], vacuum string field theory [18] and universal solutions [6]–[12]. This is because of the fact that the equation of motion with respect to Ψ_- is linear in Ψ_- owing to twist invariance of the CSFT action, and one can make a consistent truncation by setting Ψ_- zero [15]. However, at least in general, it is an nontrivial question whether an nonzero twist odd part is arrowed. If a twist odd component cannot be removed and has physical effects, such situation indicates existence of new classical vacuum without twist symmetry. Of course, such vacuum contradicts the uniqueness of the closed string vacuum, which is a part of Sen's conjecture.

Thus our questions are as follows: twist symmetry of CSFT can be violated by a twist odd part of the universal solutions? Does such vacuum exists? Is it different from the closed string vacuum? In this paper, we would like to answer such questions by considering two

[2] The twist operation, which is denoted as Ω usually, reverses an orientation of an open string according to $\sigma \to \pi - \sigma$.

JHEP10(2005)078

parameter family of the solution defined by

$$g(w) = 1 + \frac{a}{2}\left(w + \frac{1}{w}\right)^2 - ib\left(w - \frac{1}{w} + w^3 - \frac{1}{w^3}\right). \tag{1.3}$$

The answer is that all things expected in above questions *never occur*, since we can show that every solution defined in terms of (1.3) is equivalent to the twist even solution considered in [6] through a certain field redefinition. Thus whole story is consistent with twist even solutions obtained earlier, and one does not need to worry about violation of twist symmetry.

The paper is organized as follows. In section 2, after reviewing twist symmetry in CSFT, we introduce two parameter family of universal solutions which will be used throughout this paper. In particular, distribution of zeros of $g(w)$ are investigated precisely. In following two sections, we give some evidences that the solutions can become nontrivial when some zeros reach unit circle. Section 3 deals with singularity of the field redefinition operator and BRST cohomology of the modified BRST operator. In section 4, we give level truncation analysis of the CSFT expanded around the classical solutions. In section 5 we give a conformal transformation which completely maps our solutions to the twist even solution considered in [6]. With the help of this map, twist symmetry of a CSFT around our solution can be easily understood and turns out to be unbroken. Finally in section 6, we summarize our results and give some discussions.

2. Universal solution with twist odd modes

2.1 Twist symmetry in CSFT

First let us summarize some aspects of CSFT concerned with the twist symmetry of open string theory. We shall borrow most notations and discussions from [1]. The CSFT action [5] on single unstable D-25 brane is given by

$$S[\Psi] = -\frac{1}{g_o^2}\left(\frac{1}{2}\langle\Psi, Q_B\Psi\rangle + \frac{1}{3}\langle\Psi,\Psi,\Psi\rangle\right), \tag{2.1}$$

where the braket $\langle\cdots\rangle$ is the multi string product defined by the noncommutative star products [1]. Open string theory has the twist symmetry Ω which reverses an orientation of an open string worldsheet. The action of twist symmetry on the open string Hilbert space is quite simple in the oscillator formalism. For an oscillator satisfying $[L_0, \phi_n] = n\phi_n$,

$$\Omega(\phi_n) = (-1)^n \phi_n\,. \tag{2.2}$$

Note that the SL$(2,\mathbb{C})$ vacuum $|0\rangle$ has twist eigenvalue -1 as discussed in [1]. Therefore we have, for example,

$$\Omega(\phi_n\,|0\rangle) = (-1)^{n+1}\phi_n\,|0\rangle\ . \tag{2.3}$$

Since the string field can be represented by a linear combination of fock space states, (2.3) defines the action of Ω on the string field. In the language of CSFT, the twist symmetry of correlation functions of CFT is conveniently expressed as [1]

$$\langle A, B\rangle = \langle \Omega(A), \Omega(B)\rangle\ , \tag{2.4}$$

$$\langle A, B, C\rangle = (-1)^{AB+BC+CA+1}\,\langle \Omega(C), \Omega(B), \Omega(A)\rangle\ , \tag{2.5}$$

where A, B and C are arbitrary string fields. Applying (2.4) and (2.5) to (2.1), and taking into account the facts that Ψ is Grassmann odd and $[Q_B, \Omega] = 0$, we can show the twist invariance of CSFT action,

$$S[\Omega(\Psi)] = S[\Psi]\,. \tag{2.6}$$

In the following, we only consider a case in which Ψ belongs to the universal subspace [15] which consists of the matter Virasoro generators, conformal ghost oscillators and the $\mathrm{SL}(2,\mathbb{C})$ vacuum. Decompose Ψ as

$$\Psi = \Psi_+ + \Psi_-\,, \tag{2.7}$$

where Ψ_+ and Ψ_- are string fields with twist eigenvalues $+1$ and -1, respectively. Plugging (2.7) into (2.1) and using (2.6), we obtain

$$\begin{aligned} S[\Psi_+ + \Psi_-] = {}& \frac{1}{2}\langle \Psi_+, Q_B\Psi_+\rangle + \frac{1}{3}\langle \Psi_+, \Psi_+, \Psi_+\rangle + \\ & + \frac{1}{2}\langle \Psi_-, Q_B\Psi_-\rangle + \langle \Psi_+, \Psi_-, \Psi_-\rangle\,. \end{aligned} \tag{2.8}$$

Equations of motion with respect to Ψ_+ and Ψ_- are

$$Q_B\Psi_+ + \Psi_+ * \Psi_+ + \Psi_- * \Psi_- = 0\,, \tag{2.9}$$

$$Q_B\Psi_- + \Psi_+ * \Psi_- + \Psi_- * \Psi_+ = 0\,, \tag{2.10}$$

respectively. Since (2.10) is linear in Ψ_-, one can make a consistent truncation to set Ψ_- zero as proposed in [15]. However one can also find a solution with nonzero Ψ_-. In the following, we would like to argue such case.

2.2 Universal solutions

In the formalism of universal solution [6], which is exact classical solutions of CSFT outside Siegel gauge [12], it is easy to construct solutions with twist odd string fields. The solution is labeled by a function $F(w)$.

$$\Psi_0 = \left\{ Q_L(F) - C_L\left(\frac{(\partial F)^2}{1+F}\right)\right\}\mathcal{I}\,, \tag{2.11}$$

where $\mathcal{I}$ is the identity string field. $Q_L(F)$ and $C_L((\partial F)^2/(1+F))$ are integrals of the BRST current and conformal ghost $c(w)$ multiplied by each functions over the left half of an open string. To satisfy the equation of motion, the function $F(w)$ must obey $F(-1/w) = F(w)$, $F(\pm i) = 0$, and the hermiticity condition which will be discussed later. Decompose $F(w)$ as

$$F(w) = F_+(w) + F_-(w)\,, \tag{2.12}$$

where $F_+(w)$ and $F_-(w)$ are even and odd functions of w respectively. Using (2.11), one can easily confirm that setting $F_-(w) \neq 0$ adds twist odd components to (2.11).

In the rest part of this paper, we shall analyze the modified BRST operator obtained by fluctuating a string field around the solution (2.11). Such operator is defined by $Q_g\Psi = Q_B\Psi + \Psi * \Psi_0 + \Psi_0 * \Psi$, and explicitly given by

$$Q_g = Q(g) - C\left(\frac{(\partial g)^2}{g}\right), \tag{2.13}$$

where $g(w) = F(w) + 1$. $Q(g)$ and $C((\partial g)^2/g)$ are defined in the same manner as Q_L and C_L, but integrals are now taken over the whole unit circle. Again, the odd part of $g(w)$ corresponds to the contribution of twist odd components of the solution.

In terms of $g(w)$, the conditions imposed on $F(w)$ are expressed as [10],

$$g\left(-\frac{1}{w}\right) = g(w)\,, \tag{2.14}$$
$$g(\pm i) = 1\,, \tag{2.15}$$
$$g(e^{i\theta}) \geq 0\,. \tag{2.16}$$

In particular, the third condition — positivity of $g(w)$ on the unit circle — ensures hermiticity of the modified BRST operator.

2.3 Solutions with third order polynomial

As a simple example of a solution including twist odd components, we shall consider

$$g(w) = 1 + \frac{a}{2}\left(w + \frac{1}{w}\right)^2 - ib\left(w - \frac{1}{w} + w^3 - \frac{1}{w^3}\right), \tag{2.17}$$

where a and b are real parameters. With this choice, two of the three conditions imposed on $g(w)$, (2.14) and (2.15), are trivially satisfied. On the other hand, the condition (2.16) will further restrict parameters a and b to be in certain region. To impose (2.16) on (2.17), another parameterization of $g(w)$ in term of its zeros is quite useful. Such parameterization also have been played an important role in the analysis of the universal solutions [6, 7, 10, 11]. Among six zeros of (2.17), three zeros are always inside the unit disk, while the rest are outside owing to the symmetry (2.14). When (2.16) is satisfied, one of three zeros inside the unit disk becomes pure imaginary, and other two are symmetric under $z \leftrightarrow -\bar{z}$. We denote the pure imaginary zero as it, where t is real, and the pair of complex zeros as x and $-\bar{x}$. In terms of these zeros, we can rewrite (2.17) as

$$g(w) = g_x(w) \cdot g_{-\bar{x}}(w) \cdot g_{it}(w)\,, \tag{2.18}$$

where

$$g_y(w) = \frac{i}{(i-y)(i+y^{-1})}\frac{(w-y)(w+y^{-1})}{w}\,. \tag{2.19}$$

It can be easily seen that (2.18) satisfies (2.14), (2.16) and $g(i) = 1$, but does *not* obeys $g(-i) = 1$ in general. Of course, the last condition is necessary.[3] Imposing $g(-i) = 1$

[3] It reduces the number of independent parameters in (2.18) to two, which is the same number of parameters contained in (2.17).

on (2.18) gives

$$\left| x + \frac{i}{t} \right| = \sqrt{\frac{1}{t^2} - 1}\,. \tag{2.20}$$

This constraint means that x is on a circle with radius $\sqrt{1/t^2 - 1}$ and center $z = -i/t$. Such x can be parameterized as

$$x\,(t, \theta) = -\frac{i}{t} + \sqrt{\frac{1}{t^2} - 1}e^{i\theta}\,. \tag{2.21}$$

Imposing $|x| \leq 1$ on (2.21) gives

$$|\cos\theta| \leq |t|\,. \tag{2.22}$$

For simplicity, consider a case where t is positive.[4] For fixed t, the complex zeros moves inside lower half of the unit disk along the circle defined by (2.21). In particular,

- We exclude $t = 1$ case, because in this case the Laurent coefficients of (2.18) diverge, therefore $g(w)$ no longer gives an well-defined BRST operator.

- The complex zeros reach to the unit circle when $\cos\theta = t$. On the other hand, the pure imaginary zero is always inside the unit disk because $0 \leq t < 1$.

Thus, the distribution of zeros is completely specified by the two parameter family of pure imaginary and complex zeros given by (2.21) within the range of parameters $0 \leq t < 1$ and $0 \leq \cos\theta \leq t$.

3. Field redefinition and cohomology

Our conjecture in this and next section is that the classical solution given by (2.18) becomes nontrivial if and only if the complex zeros (x and $-\bar{x}$) reach to the unit circle.[5] Using analytic methods, we will give two evidences for our conjecture. Further evidence from numerical analysis will be given in section 4.

3.1 Singularity of the field redefinition

First evidence for our conjecture can be obtained by an analysis of field redefinitions associated with the modified BRST operator. In the same manner as in [6, 7, 10, 11], the modified BRST operator can be formally transformed to Q_B as

$$Q_g = e^{q(h)} Q_B e^{-q(h)}\,, \tag{3.1}$$

where $h(w) = \log g(w)$ and $q(h)$ is defined by

$$q(h) = \oint dw\, h(w) j_{gh}(w)\,, \tag{3.2}$$

[4] Negative t case can be obtained by reversing all zeros in positive case by $z \to \bar{z}$.
[5] It is well known that similar results for twist even cases hold [6, 10, 11].

where $j_{gh}(w)$ is the ghost number current. When the operator $e^{q(h)}$ is regular, (3.1) defines well-defined transformation; we can transform the CSFT action with Q_g into the original action with Q_B by a field redefinition $\Psi \to e^{-q(h)}\Psi$. In such case, a classical solution has no physical meaning and corresponds to gauge degree of freedom. On the other hand, if $e^{q(h)}$ happens to be singular, we cannot perform such field redefinition. In such case a solution is expected to be nontrivial.

In order to evaluate such singularity, we shall use an oscillator expansion of $q(h)$. We can obtain this using (2.18) and (2.19) and the fact that $|x| \leq 1$ and $|t| < 1$. The result is

$$q(h) = -2\left\{\log\left(|x-i|^2(1-t)\right)\right\} q_0 - \sum_{n=1}^{\infty} \frac{\mathcal{A}_n(t)+\mathcal{B}_n(x)}{n} \left(q_n + (-1)^n q_{-n}\right), \tag{3.3}$$

where

$$\begin{aligned} \mathcal{A}_n(t) &= (-it)^n\,, \\ \mathcal{B}_n(x) &= (-x)^n + \bar{x}^n\,, \end{aligned} \tag{3.4}$$

and q_n is the oscillator mode of $j_{gh}(w)$. Using (3.3), we can evaluate the singularity of $e^{q(h)}$ by performing normal ordering with respect to the SL$(2,\mathbb{C})$ vacuum. A calculation is easily performed by using $[q_n, q_m] = n\delta_{n+m}$, and it turns out that a contraction of two zeros y and z contributes a factor $(1-yz)^{-1/2}$. A potentially divergent factor comes from a contraction between x and $-\bar{x}$ parts in $q(h)$ and it amounts to

$$(1-|x|^2)^{-1}\,. \tag{3.5}$$

In fact, it diverges when x reaches to the unit circle. Thus, in this case, the classical solution is expected to be nontrivial, since CSFT action with Q_g no longer equivalent to the original action.

3.2 Cohomology

As a further evidence for our conjecture, we shall show that the cohomology of the modified BRST operator vanishes when complex zeros come to unit circle. This means no physical excitation of open string around the solution. Such result is expected according to to Sen's conjecture, if our nontrivial solutions correspond to the closed string vacuum. The proof can be done in a way similar to [7, 11], where the cohomology of the BRST operator is mapped to the modified Kato-Ogawa cohomology [19] with 'wrong' ghost numbers.

For a critical configuration satisfying $|x_c| = 1$ and $|t_c| < 1$, (2.18) becomes

$$g_{x_c,t_c}(w) = \frac{-it_c}{|x_c-i|^4(1-t_c)^2} \frac{(w-x_c)^2(w+\bar{x}_c)^2(w-it_c)(w-it_c^{-1})}{w^3}\,. \tag{3.6}$$

A crucial step is rewriting (3.6) to

$$g_{x_c,t_c}(w) = \frac{1}{|x_c-i|^4(1-t_c)^2} \times w^2 \times e^{h_{x_c}(w)} \times e^{h_{t_c}(w)}\,, \tag{3.7}$$

where

$$h_{x_c}(w) = -2\sum_{n=1}^{\infty}\left\{\frac{x_c^n+(-\bar{x}_c)^n}{n}\right\}w^{-n},$$
$$h_{t_c}(w) = -\sum_{n=1}^{\infty}\frac{(-it_c)^n}{n}\left(w^n+(-1)^n w^{-n}\right). \tag{3.8}$$

Next, using the formula $Q_{e^h g} = e^{q(h)}Q_g e^{-q(h)}$ [6], we can rewrite (3.7) into corresponding operator equation,

$$Q_g = \frac{1}{|x_c-i|^4(1-t_c)^2}e^{q(h_{x_c})}e^{q(h_{t_c})}Q_B^{(2)}e^{-q(h_{x_c})}e^{-q(h_{t_c})}, \tag{3.9}$$

where $Q_B^{(2)} = Q_2 - 2c_2$ is the 'shifted' BRST operator obtained by applying a replacement $c_n \to c_{n+2}, b_n \to b_{n-2}$ to Q_B [7]. Note that the transformations in (3.9) is regular, because $|t_c| < 1$ and $q(h_{x_c})$ consists of only negative frequency modes of the ghost number current. The cohomology of $Q_B^{(2)}$ obtained in [7] mapped into that of Q_g as

$$|\Psi_{\rm phys}\rangle = e^{q(h_{x_c})}e^{q(h_{t_c})}\left(|P\rangle\otimes b_{-2}\,|0\rangle + |P'\rangle\otimes|0\rangle\right), \tag{3.10}$$

where $|P\rangle$ and $|P'\rangle$ are the DDF states [20]. Since a ghost number of this state is not equal to unity, the state is actually zero in the context of classical CSFT without gauge fixing.[6] Thus we have obtained a vanishing cohomology.

One the other hand, for trivial solutions whose all zeros are inside the unit disk, it is easily shown that the BRST cohomology is equivalent to the Kato-Ogawa cohomology.[7] Therefore, our result implies that the solutions with complex zeros on the unit circle are nontrivial one corresponding to the closed string vacuum.

4. Level truncation analysis

As another nontrivial test for our proposal, we can perform the level truncation analysis of the CSFT around the classical solutions. An analysis can be done in almost same manner as [11, 8], except for the fact that we must include string fields of odd levels. In Siegel gauge, the 'normalized' CSFT potential around the classical solution becomes

$$f_g(\Psi) = 2\pi^2\left(\frac{1}{2}\langle\Psi, c_0 L_g\Psi\rangle + \frac{1}{3}\langle\Psi,\Psi,\Psi\rangle\right). \tag{4.1}$$

For the representation of $g(w)$ given in (2.17), the gauge fixed kinetic operator L_g is given by

$$L_g = \{Q_g, b_0\}$$
$$= (1+a)L_0 + \frac{a}{2}\left(L'_2+L'_{-2}\right) - ib\left(L'_1 - L'_{-1} + L'_3 - L'_{-3}\right) - \kappa(g), \tag{4.2}$$

[6] In this theory, only ghost number one fields are allowed.

[7] This can be shown by mapping the Kato-Ogawa cohomology by $e^{q(h)}$ which appearers in (3.1).

where $L'_n = L_n + nq_n + \delta_{n,0}$ is the $c = 24$ twisted Virasoro generator and $\kappa(g)$ is a real valued integral defined by

$$\kappa(g) = \oint \frac{dw}{2\pi i} \frac{(g'(w))^2}{g(w)} \times w\,. \tag{4.3}$$

Following [8, 11], let us consider a local minimum of the potential. As explained in [8], if our solution correctly reproduce the D-brane tension and obeys Sen's conjecture, then the value of the potential at local minimum should depends on the positions of complex zeros as

$$f_g(\Psi_0) = \begin{cases} -1 & (|x| < 1)\,, \\ 0 & (|x| = 1)\,, \end{cases} \tag{4.4}$$

where Ψ_0 is the local minimum. According to (4.4), a plot of $f_g(\Psi_0)$ with respect to parameters of $g(w)$ is expected to have discontinuities when $|x| = 1$. Of course, an actual plot obtained by level truncation will never coincide with (4.4) exactly, though it will approach (4.4) as level increases.

The only difference between our analysis and [11, 8] is inclusion of odd level fields. This is necessary for our case since the potential (4.1) is no longer invariant under Ω because L_g does not commutes with Ω in general. The gauge fixed equations of motion obtained from (4.1) become

$$c_0 L_{g,+}\Psi_+ + c_0 L_{g,-}\Psi_- + \Psi_+ * \Psi_+ + \Psi_- * \Psi_- = 0\,, \tag{4.5}$$
$$c_0 L_{g,+}\Psi_- + c_0 L_{g,-}\Psi_+ + \Psi_+ * \Psi_- + \Psi_- * \Psi_+ = 0\,, \tag{4.6}$$

where $L_{g,+}$ and $L_{g,-}$ are twist even and odd parts of (4.2), respectively. Note that Ψ_- cannot be set to zero to satisfy (4.6) because of the presence of the odd part of the kinetic operator.

Now we shall try to find a level truncated solution of (4.5) and (4.6) in the universal subspace [15]. Actual calculation is done as follows:

- Required techniques to perform level truncation analysis have already developed by many authors, and they can also be applied for our case. To compute three string vertex, we use the conservation current method and an algorithm used in [16].

- We can easily see that there are no level 1 fields in Siegel gauge. Odd fields appearers first at level 3. That is given by,

  $$|\Psi_{\text{level }3}\rangle = is_1 c_{-2}\,|0\rangle + is_2 b_{-2}c_{-1}c_1\,|0\rangle + is_3 L^{(X)}_{-3} c_1\,|0\rangle\ , \tag{4.7}$$

  where s_1, s_2 and s_3 are real parameters.

- We have done all computations up to level (6, 18).

Since our solution has two independent parameter, we must fix one of them to draw a two dimensional plot of potential values. We will do this in two different ways.

level	t	s_3	s_1	$f_a(\Psi_0)$
(3, 9)	0.5161	-0.0168	0.0429	-0.9111
(4, 12)	0.5298	-0.0181	0.05119	-0.9812
(5, 15)	0.5260	-0.0182	0.0541	-0.9747
(6, 18)	0.5259	-0.0187	0.0543	-0.9928

Table 1: Value of some string fields at $a = 0.1$.

4.1 $a = b$ case

As first example of one parameter solution, we set $a = b$ in (2.17). This gives

$$g(w) = 1 + \frac{a}{2}\left(w + \frac{1}{w}\right)^2 - ia\left(w - \frac{1}{w} + w^3 - \frac{1}{w^3}\right). \tag{4.8}$$

The condition (2.16) determines the range of a as

$$-\frac{2}{9} \leq a \leq \frac{27}{50}. \tag{4.9}$$

Two boundaries of the above region correspond to critical points where the complex zeros are on the unit circle. Positions of zeros at two critical points are given by

$$(x_c, it_c) = \begin{cases} \left(\frac{\sqrt{3}+i}{2}, -\frac{1}{2}i\right) & \left(a = -\frac{2}{9}\right) \\ \left(\frac{\sqrt{5}-2i}{3}, \frac{2}{3}i\right) & \left(a = -\frac{27}{50}\right). \end{cases} \tag{4.10}$$

According to (4.4), the potential value will depend on a as

$$f_a(\Psi_0) = \begin{cases} -1 & \left(-\frac{2}{9} < a < \frac{27}{50}\right), \\ 0 & \left(a = -\frac{2}{9}, \frac{27}{50}\right). \end{cases} \tag{4.11}$$

Using (4.8), we can compute the value of the potential at a local minimum $f_a(\Psi_0)$ as a function of a. Figure 1 is a plot of our result up to level 6. We can see that a flat region at the bottom glows, and two flat regions at both sides shrink as level increases. Surely curves approach the discontinuous configuration predicted in (4.11). In addition, we can see that level 3 fields — first odd fields — surely contribute to the potential value and improve level 2 result.

Table 1 picks up values of some fields at a local minimum in the case of $a = 0.1$. t is the level 0 field, and s_1 and s_3 are level 3 fields appeared in (4.7). Indeed, we can see that level 3 fields acquire nonzero values as expected.

4.2 Fixed t case

Next we consider another one parameter family of our solutions obtained by fixing a position of pure imaginary zero appearers in (2.18). The relation between zeros x and t is already

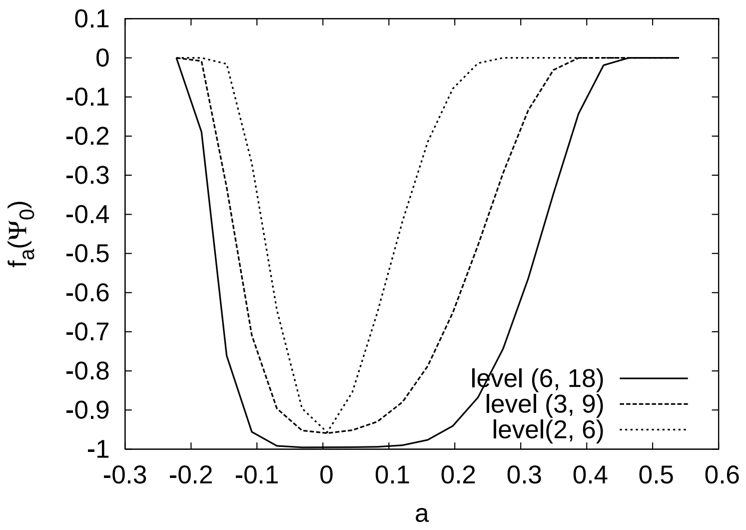

Figure 1: A plot of potential value of $a = b$ solution

given in (2.21). By fixing t at some value t_0, we have

$$x(\theta) = -\frac{i}{t_0} + \sqrt{\frac{1}{t_0^2} - 1}\, e^{i\theta}\,. \tag{4.12}$$

In order to $x(\theta)$ to be inside unit disk, we must restrict θ to $0 \leq \cos\theta \leq t_0$. Thus (2.18) and (4.12) define one parameter family of solution for each t_0 with parameter θ. $x(\theta)$ reaches the unit circle when $\cos\theta = t_0$, so this value of θ corresponds to critical configuration where solution expected to be nontrivial. Again, according to (4.4), the parameter dependence of the potential value will be

$$f_\theta(\Psi_0) = \begin{cases} -1 & (\cos\theta < t_0)\,, \\ 0 & (\cos\theta = t_0)\,. \end{cases} \tag{4.13}$$

Now, we can perform level truncation analysis for various values of t in the same manner as in section 4.1. Figures in 2 are plots of the potential value at $t = 0.3$ and $t = 0.7$ respectively. Again, curves in the plots approach to desired one as level increases. In both plots, a flat region at the left bottom of the curves grows, and another flat region at the right top becomes narrower as level goes higher.

5. Twist symmetry around classical solutions

The CSFT action expanded around our solution is no longer invariant under the twist operation Ω in general. For nonzero b, (2.17) has odd part, so Q_g includes twist odd components. Consequently, Ω no longer commutes with Q_g. From this fact, we can see

$$S_g[\Omega(\Psi)] \neq S_g[\Psi]\,, \tag{5.1}$$

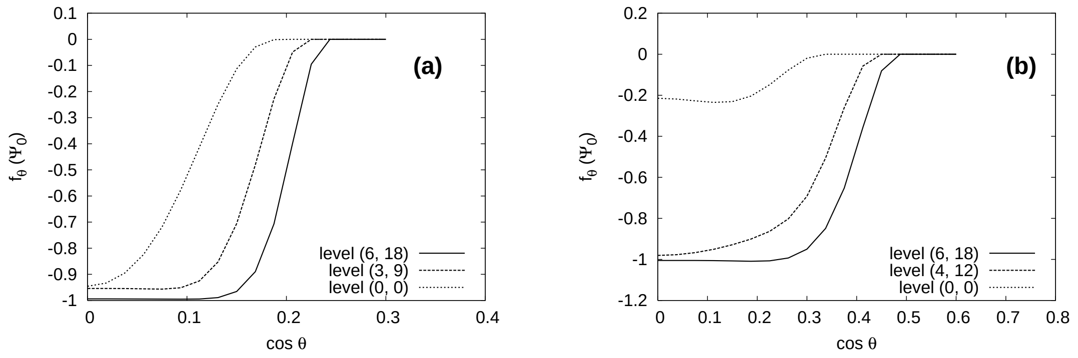

Figure 2: Plots of the potential values at (a) $t = 0.3$ and (b) $t = 0.7$ with respect to $\cos\theta$

where $S_g[\Psi] = 1/2\,\langle\Psi, Q_g\Psi\rangle + 1/3\,\langle\Psi,\Psi,\Psi\rangle$. An naive interpretation of the above fact is that the twist symmetry of original CSFT is broken at the classical vacuum specified by one of our solution. However, at least for a trivial solution, such phenomenon never occur, since a CSFT around a trivial solution is equivalent to the original CSFT described by Q_B, which has twist invariant action. Therefore the twist symmetry never be violated in a CSFT around a trivial solution. This fact can be easily confirmed by mapping the twist operator Ω to new one.

Consider the function $g(w)$ such like (2.17) whose zeros are inside the unit disk. Then the CSFT action around the solution associated with $g(w)$ is described by the modified BRST operator Q_g. In this case, one can always find a regular transformation[8] which satisfy

$$UQ_gU^{-1} = Q_B\,. \tag{5.2}$$

Using U, we can construct the deformed twist operator

$$\widetilde{\Omega} = U\Omega U^{-1}\,. \tag{5.3}$$

Since U is regular, $\widetilde{\Omega}$ has same algebraic properties as the original twist. Furthermore, $\widetilde{\Omega}$ leaves $S_{Q_g}[\Psi]$ invariant. Therefore, twist invariance of the SFT with Q_g is represented by $\widetilde{\Omega}$.

On the other hand, for nontrivial solutions whose some zeros of $g(w)$ are on the unit circle, a situation is quite different. An attempt to find regular transformation like (5.2) always fails. This is because CSFT around nontrivial solution is no longer equivalent to the original theory with Q_B. Thus we cannot use (5.2) to obtain new twist operator.

However, if there exists another twist even BRST operator which can be connected smoothly to Q_g, similar argument to the case of trivial solution is possible. We would like to try to find a regular transformation and a twist even BRST operator which satisfy

$$UQ_gU^{-1} = Q_{\tilde{g}}\,, \tag{5.4}$$

[8] For example, one can set $U_g = e^{q(\log g)}$. Also U can be taken to be a conformal transformation generated by $K_n = L_n - (-1)^n L_{-n}$

where $\tilde{g}(w)$ is an *even* function whose some zeros are on the unit circle. Of course, $Q_{\tilde{g}}$ becomes twist even operator associated with nontrivial solution. Once U is find, new twist operator which leaves $S_g[\Psi]$ invariant can be constructed in the same way as the case of trivial solutions.

To find regular transformation in (5.4) , let us consider one parameter family of nontrivial solutions whose the complex zeros are *always* on the unit circle. Setting $\cos\theta = t$ in (2.21) realize such situation. With this choice, x becomes

$$x = \sqrt{1-t^2} - it\,. \tag{5.5}$$

Inserting (5.5) into (2.18), we find

$$g_t(w) = 1 + \frac{a(t)}{2}\left(w + \frac{1}{w}\right)^2 - ib(t)\left(w - \frac{1}{w} + w^3 - \frac{1}{w^3}\right), \tag{5.6}$$

where

$$a(t) = \frac{3t^2-1}{2(1-t^2)^2}\,, \qquad b(t) = \frac{t}{4(1-t^2)^2}\,. \tag{5.7}$$

Note that $b(t)$ vanishes at $t = 0$. This means that the BRST operator becomes twist even at $t = 0$. In fact, this case is nothing but the nontrivial Takahashi-Tanimoto solution [6] which is given by

$$g_{TT}(w) = g_0(w) = -\frac{1}{4}\left(w + \frac{1}{w}\right)^2\,. \tag{5.8}$$

Following (5.4), let us try to find $U(t)$ satisfying

$$U(t)Q_tU(t)^{-1} = Q_0\,, \tag{5.9}$$

where Q_t denotes Q_{g_t}.[9] It turns out that an attempt to find $U(t)$ in terms of ghost current transformation fails. Thus we try to find $U(t)$ in terms of conformal transformation. By a formal argument of contour integrals of the BRST currents, (5.9) can be reduced to the equation

$$g_t(f_t^{-1}(w)) = g_0(w)\,, \tag{5.10}$$

where $f_t(w)$ is the finite conformal map generated by $U(t)$. Though this equation can be solved, it turns out that the generator of $U(t)$ becomes an infinite sum of Virasoro generators $K_n = L_n - (-1)^nL_{-n}$. This makes it difficult to see whether $U(t)$ is regular. Instead of dealing with this transformation, let us consider more simpler one. Our ansatz is

$$g_t(f_t^{-1}(w)) = g_0(w) \times \sigma(w)\,, \tag{5.11}$$

where $\sigma(w)$ has no poles or zeros on the unit circle. With such choice, $\sigma(w)$ can be absorbed into regular field redefinition and has no physical effect. In order to archive (5.11), $f_t(w)$ must map zeros of $g_t(w)$ on the unit circle to that of $g_0(w)$. From (5.5), this condition is explicitly represented as

$$f_t(x(t)) = 1, \quad f_t(-\bar{x}(t)) = -1\,. \tag{5.12}$$

[9]Note that Q_0 is the nontrivial Takahashi-Tanimoto BRST operator.

It turns out that $\mathrm{SL}(2,\mathbb{C})$ is enough to satisfy the above condition. From (5.5), we can obtain such conformal map satisfying (5.12):

$$f_t(w) = \frac{A(t)w + B(t)}{-B(t)w + A(t)}, \tag{5.13}$$

where

$$A(t) = \cosh\left(\frac{1}{2}\tanh^{-1} t\right), \qquad B(t) = i\sinh\left(\frac{1}{2}\tanh^{-1} t\right). \tag{5.14}$$

Finally, using the explicit expression (5.6) and (5.7), and plugging (5.13) into (5.11), we have

$$\sigma(w) = -4w^2 \frac{\left\{(t^3+3t)w^2 - 2i(3t^2+1)w - (t^3+3t)\right\}}{(tw^2 - 2iw - t)^3}. \tag{5.15}$$

Indeed, it is easily confirmed that $\sigma(w)$ has no poles or zeros on the unit circle when $0 \leq t < 1$. Now that $\sigma(w)$ turns out to give regular field redefinition, the decomposition (5.11) can be rewritten into operator equation. This gives our final formula

$$U(t)Q_t U(t)^{-1} = e^{q(\Sigma)} Q_0 e^{-q(\Sigma)}, \tag{5.16}$$

where $\Sigma(w) = \log\sigma(w)$ and

$$U(t) = \exp\left\{\frac{i}{2}\left(\tanh^{-1} t\right) K_1\right\}. \tag{5.17}$$

As desired, both $U(t)$ and $e^{q(\Sigma)}$ are regular transformations. Therefore, Q_t is transformed into twist even operator Q_0 by a regular transformation. New twist operator which leaves $S_{g_t}[\Psi]$ invariant is

$$\Omega' = e^{-q(\Sigma)} U(t) \Omega U(t)^{-1} e^{q(\Sigma)}. \tag{5.18}$$

Thus we have shown that nontrivial solutions with twist odd modes considered in this paper is equivalent to the twist even nontrivial solution specified by $g_0(w)$.

6. Conclusions and discussions

In this paper, we have constructed a simple example of classical solutions of CSFT which include twist odd modes. A detailed analysis of the meromorphic function which governs the solution shows that our solutions become nontrivial when complex zeros of the function reach the unit circle.

Furthermore, it was shown that the BRST cohomology of the kinetic operator of CSFT expanded around the nontrivial solution is equivalent to that of the twist even solution given in [6]. Also we have performed level truncation analysis using the CSFT potential obtained by expanding original CSFT action around our solutions. Our plots of the value of the CSFT potential with respect to positions of the complex zeros indicate that our solutions are consistent with Sen's conjecture and nontrivial solutions correspond to the closed string vacuum.

Finally, we have shown that our nontrivial solutions are equivalent to the twist even solutions found in [6] by constructing a conformal map which connects BRST operators associated with each solution. With the help of this map, the twist operator of CSFT expanded around our solutions was obtained from ordinary twist operator Ω. Thus, our result suggests that the twist symmetry is unbroken even for our solutions which contain twist odd modes, and supports the uniqueness of the closed string vacuum.

While our analysis is done in the classical CSFT where no gauge fixing condition is imposed, it is interesting to consider same situation as ours, i.e., classical solutions involving twist odd modes in the CSFT in Siegel gauge. We have found that there are no twist odd excitations which reproduce the value of the potential close to the D-25 brane tension, at least up to level 3. It would be nice if there is general arguments whether nonzero values of twist odd fields are possible.

It is also interesting to consider more general solutions than the case considered in this paper. If we limit ourself to the polynomial case, a typical form of the function $g(w)$ will be

$$g(w) = \sum_{n=0}^{N} a_n (w^n + (-1)^n w^{-n}) \,. \tag{6.1}$$

The most crucial point is to specify the distribution of zeros under three conditions imposed on $g(w)$. If this becomes clear, it is straightforward to classify all solutions in this class.

Our results and earlier works [6]–[13] are enough to expect that all nontrivial solutions belong to the class of (6.1) are equivalent, thus correspond to the unique solution i.e., closed string vacuum. Such consideration of the moduli space of SFT will give us deep insights into the nature of SFT and string theory.

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